velocity moments only and is not a "white-noise" type of forcing function. This explicit and rather marked dependence of the correlation structure on the third-order moments (third-order moments are sometimes called "turbulence self-interaction") is what separates turbulence from other naturally occurring random phenomena and also what conveys upon it, its most interesting feature. Indeed, without this dependence, the study of turbulence reduces to that of classical (Gaussian) diffusion and it has already been established (see the introduction) that such analyses do not produce satisfactory, or even acceptable, results.

The solution to Eq. (6) is[4]

$$\chi(t) = e^{-\int\alpha(t)\mathrm{d}(t)}\left[\int e^{\int\alpha(t)\mathrm{d}t}E(t)\mathrm{d}t + C\right] \tag{7}$$

where C is a constant determined from initial conditions. Equation (7) establishes the role of third-order velocity moments in the time dependence of $\chi(t) = \sigma^2(t)\chi(t)$ and also their intimate relationship with the "viscous-friction factor," $\alpha(t) = 8\nu/\lambda^2$. Without them, i.e., when $E(t) \simeq 0$, the solution to Eq. (6) is simply

$$\chi(t) = Ce^{-\int\alpha(t)\mathrm{d}t} \tag{8}$$

a result with no obviously interesting characteristics. In contradistinction, Eq. (7) exhibits a feature bearing further investigation. To be sure, if $\alpha(t)$ is negligibly small (viz., "high Reynold number" turbulence), $\chi(t)$ then reduces to $\chi(t) \simeq \int E(t)\mathrm{d}t + C$; If $E(t)$ is also some constant,[5] say E_0, the solution becomes

$$\chi(t) \simeq E_0 t + C \tag{9}$$

Equation (9) suggests that, under appropriate conditions, rapid increases in turbulence intensity, e.g., $\mathrm{d}[\sigma^2(t)]/\mathrm{d}t \sim t$, are automatically accompanied by decreases in the integral scale. The overall result of this somewhat typical combination is a turbulence that is highly random and has high intensity ("severe" turbulence), while the completely reverse process produces a relatively "moderate" turbulence. Through this type of analysis, the potential role of third-order velocity moments in augmenting, or perhaps even themselves producing, the "patchy" structure of turbulence, where periods of intense turbulence are followed by relatively quiet periods, is undeniably established.

Application to an Airplane Flying through "Frozen" Nonhomogeneous Turbulence

For an airplane flying along a horizontal path (i.e., in the x direction through "frozen" *nonhomogeneous turbulence*, the turbulence equations of motion with respect to a fixed (ground-based) set of axes are

$$\frac{\partial(U_iU_l)}{\partial x_l} = \nu\frac{\partial^2 U_i}{\partial x_l \partial x_l} \tag{10}$$

where the pressure gradient terms have been neglected. Taking the mean flow velocity to be zero, or at the very least "small" in comparison to the forward flight speed U of the aircraft, the Navier-Stokes equation with respect to the aircraft set of axes reduces to

$$-U\frac{\partial u_i}{\partial x} + \frac{\partial(u_iu_l)}{\partial x_l} = \nu\frac{\partial^2 u_i}{\partial x_l \partial x_l} \tag{11}$$

Here the "convection" term $(-U\partial u_i/\partial x)$ rather naturally converts into $(\partial u_i/\partial t)$, i.e., $[-U(\partial u_i/\partial x)] \sim (\partial u_i/\partial t)$, and ultimately all correlations in the two-point stochastic-averaged equation convert analogously, viz,

$$C_{ij}(x,r) = \langle u_i(x)u_j(x+r)\rangle \sim C_{ij}(t,r) \tag{12}$$

$$S_{ilj}(x,r) = \langle u_i(x)u_l(x)u_j(x+r)\rangle \sim S_{ilj}(t,r) \tag{13}$$

etc. In this way, the "frozen" nonhomogeneous turbulence problem, where all correlations are functions of x (and r), is brought into line with the fundamental problem of homogeneous isotropic "nonfrozen" turbulence, where all correlations are functions of time. Equation (11), with $[-U(\partial u_i/\partial x)]$ replaced by $(\partial u_i/\partial t)$, is the equation from which the Kármán-Howarth equation, Eq. (1), is under appropriate constraints eventually derived (Ref. 3, p. 100).

Conclusions

Third-order velocity product moments are indeed important in the correlation structure of turbulence. This result requires that these moments be explicitly included in any faithful simulation of same. These moments ingress as a "driving force" into the differential equation for the two-point velocity correlation of the turbulence and as much convey upon it, its characteristically unique features. Without them, the dynamical aspects of turbulence reduce to those of Gaussian diffusion and unjustly purloin from it its natural "self-interaction," as well as its interaction with the underlying mean flow. Recall that these "interactions," arise from the nonlinear convective term in the Navier-Stokes equation. Under select circumstances, third-order moments also reinforce the "patchy" nature of turbulence.

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Experimental and Theoretical Study of Wings with Blunt Trailing Edges

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Introduction

WINGS in subsonic flow are characterized by a rounded leading edge and a sharp trailing edge. Currently, considerable research work is being done to increase the lift by cir-

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culation control on wings, CCW, (see Refs. 1 for a bibliography on this topic). On such wings, the trailing edge is not sharp; rather, it is blunt with some rounded portions to facilitate blowing from the trailing edge. The compressor and turbine blades also have blunt trailing edges to avoid the loss of efficiency due to erosion on sharp trailing edges. From a fabrication point of view, airfoils with blunt trailing edges are easier to make. Hence, there is justification to study airfoils with blunt trailing edges.

Although some information is available on the effects of a blunt trailing edge on the aerodynamic characteristics,[2-5] there is some doubt about the optimum thickness of the trailing edge. Hoerner[2] has studied a 40% thick airfoil of aspect ratio 4 with a blunt trailing edge. At a Reynolds number, based on airfoil chord R_c of 10^6, he found that the lift coefficient increased considerably compared to a wing with sharp trailing edge. Smith and Schaefer[3] have measured lift, drag and pitching moment on three airfoil sections formed by removing 1.5, 4 and 12.5% of the original chord from trailing edge of an NACA 0012 airfoil section at R_c of 3×10^6 and 6×10^6. Their results indicate a small change in lift coefficient, but some increase in drag coefficient. Tanner[4] finds a 10% increase in the maximum lift coefficient for a wing with thick trailing edge. He also finds that the increase in drag can be reduced by suitable castellations in the trailing edge along the span. Nash[5] has discussed the effect of trailing-edge blunting at supersonic speeds.

In this Note, rectangular wings with NACA 0012 airfoils are studied experimentally and theoretically. The bluntness is obtained by progressively cutting the chord of the basic airfoil, with sharp trailing edge, at distances of 5, 10, 15, and 20% of the chord from trailing edge. In the theoretical investigation the cutoff length extends up to 50% of chord. In the experimental investigation, the lift L and drag D are obtained from wind tunnel balance measurements at angles of attack α from 0-20 deg at R_c of 4×10^5 for the basic airfoil. The theoretical results are obtained using surface vorticity technique.

Wing Models and Experimental Setup

Seasoned teak wood is used to make the five rectangular wings of 1 m span. The profile used is the NACA 0012 and the chord is 0.2 m for the sharp trailing-edged wing. For the other four wings, with blunt trailing edges, the profile was cut off progressively from the trailing-edge end. The chords of these blunt wings are 0.19, 0.18, 0.17, and 0.16 m (Fig. 1); the thickness at the trailing edge for these wings is given in Table 1. For convenience, the wing with sharp trailing edge is designated as wing I and the others obtained by cutting of the chord by 5, 10, 15, and 20% are designated as wings II, III, IV, and V, respectively (Fig. 1). The aspect ratios A for these wings are 5, 5.27, 5.56, 5.88, and 6.25, respectively.

Experiments were conducted in a closed-circuit open-jet wind tunnel with 1.5 m diameter test section. The speed in the tunnel can be varied continuously. Each of the five wing models was attached to a semiautomatic mechanical six-component wire balance. Two suspension points were symmetrically located on the quarter-chord line of the wing at a distance of 0.4 m apart and the third suspension point is 0.4 m from the center of the front suspension points (Fig.2). The alignment of the wing models was such that all the three suspension points are in the same horizontal plane and all of the points on the leading edge of the wing are equidistant from the exit plane of the wind tunnel nozzle.

All the five models were tested at a speed U of 32 m/s. The Reynolds number, based on the chord c of 0.2 m is 4×10^5, for wing I. For wings II, III, IV, and V it is 3.8×10^5, 3.6×10^5, 3.4×10^5, and 3.2×10^6, respectively. It is presumed that the lift coefficient C_L and drag coefficient C_D would change only negligibly when R_c changes from 3.2×10^5 to 4×10^5. The angle of attack is varied in steps of 2 deg from $\alpha=0$-12 deg and then in steps of 1 deg up to $\alpha=20$ deg.

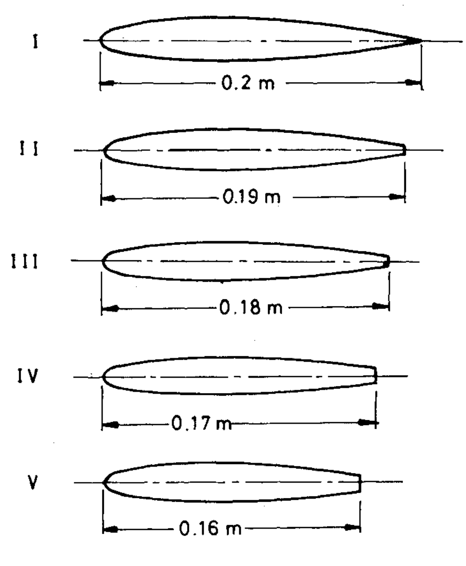

Fig. 1 Airfoil sections for wings.

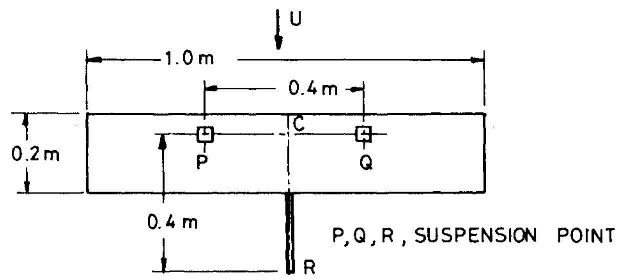

Fig. 2 Wing model and suspension points.

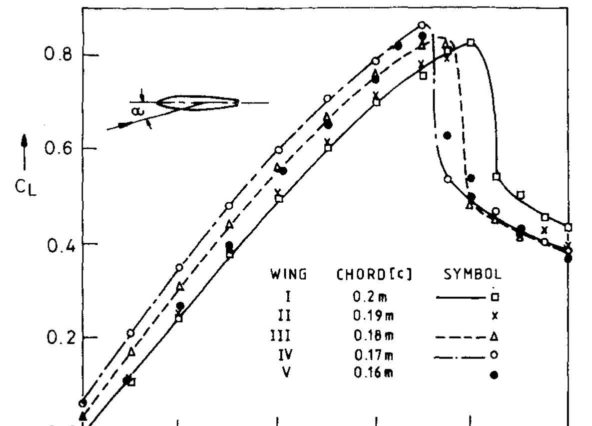

Fig. 3 Variation of C_L with α.

Experimental Results

The lift coefficient $C_L=L/(\frac{1}{2}\rho U^2 S)$ is based on the actual area S of the wing. The variation of C_L with angle of attack α is shown in Fig. 3 for all wings. For wings I and II, the C_L values at a given α are nearly the same. However, for wings III and IV, the values of C_L are higher than those for wing I in the range of $\alpha=$ 0-14 deg. The values of C_L for wing V are lower than those for wing III and IV, in the range of $\alpha=0$-12 deg. The values of maximum lift coefficient $C_{L_{max}}$, slope of lift curve $dC_L/d\alpha$, and angle of stall α_{stall} are presented for all wings in Table 1. It is seen that $dC_L/d\alpha$ increases, initially, as

Table 1 Comparison of wings

Parameter	I	II	III	IV	V
Wing chord, m	0.2	0.19	0.18	0.17	0.16
Thickness at trailing edge, mm	0	3.2	5.8	8.2	10.5
$dC_L/d\alpha$ per degree (exp.)	0.0634	0.0638	0.0674	0.0680	0.0650
$dC_L/d\alpha$ per degree (theory)	0.0824	0.0843	0.0863	0.0883	0.0907
$(dC_L/d\alpha)_0$ per degree (theory) (for airfoils)	0.1195	0.1208	0.1222	0.1237	0.1250
$C_{L\max}$	0.825	0.83	0.835	0.865	0.84
α_{stall}, deg	16	16	15	14	14
$C_{D\min}$	0.0065	0.0065	0.011	0.016	0.021
$(L/D)_{\max}$	22	21	17	16.1	13.5

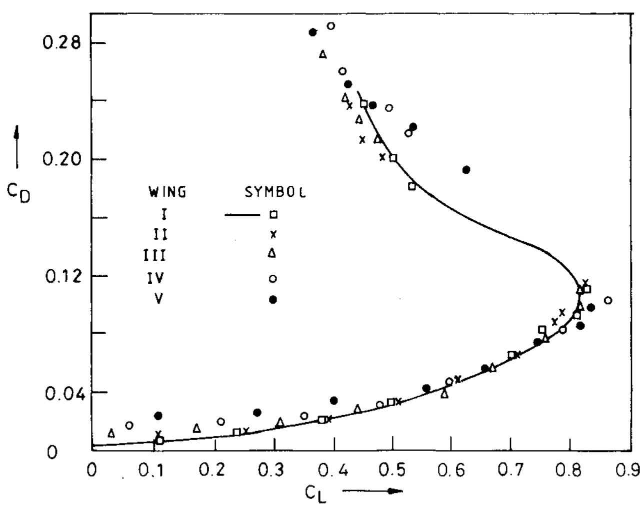

Fig. 4 Variation of C_D with C_L.

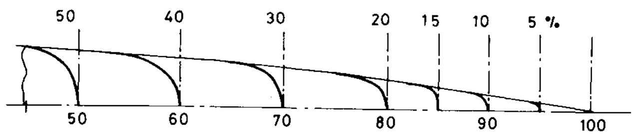

Fig. 5 Trailing-edge rounding-off for computation.

the trailing edge becomes more and more blunt but reaches a maximum and then decreases. Baskaran and Holla[6] have found theoretically that the lift on an airfoil with a blunt trailing edge is very sensitive to the location of the rear stagnation point. Figure 3 indicates that this shift may take place at all angles of attack from 0-12 deg and that the shift depends on the bluntness of the trailing edge. The maximum lift coefficient is also the highest for wing IV. But the increase in $C_{L\max}$, as compared to wing I, is not large as α_{stall} is lower for wing IV.

Flow visualization studies were carried out in a water channel on airfoil sections used on wings I and IV at $R_c = 10^4$. At angles of attack up to 12 deg the airfoil of wing IV showed a smaller extent of separated region than that on the airfoil of wing I. This seems to confirm wind tunnel test results where higher C_L values were obtained for wing IV than for wing I. No definite conclusions could be drawn about the shift of the rear stagnation point from these flow visualtion studies.

To obtain the drag D of the wing, the tare drag of the suspension is subtracted from the measured values of total drag. The drag coefficient C_D is calculated from $C_D = D/(\frac{1}{2}\rho V^2 S)$. Figure 4 shows C_D vs C_L for all five wings.

It is seen that the effect of the bluntness is to increase C_D at low values of C_L (also see Table 1). This is due to the higher base pressure drag for wings with blunt trailing edges. At higher values of lift coefficient ($C_L > 0.5$), there is little difference in the values of C_D for various wings. This is due to the fact that, at these values of C_L, the flow separates significantly ahead of the trailing edge and hence the shape of the trailing edge does not appreciably influence C_D. It is interesting to note that cutting off the trailing edge by 5% does not increase the drag appreciably as compared to a wing with a sharp trailing edge.

The variation of aerodynamic efficiency (L/D ratio) is also given in Table 1. The maximum value occurs around $\alpha = 4$ deg. For wing I, the maximum value is 22 and for wing IV 16.1. At higher angles of attack, viz. at $\alpha = 14$ deg, all wings had almost the same value of L/D (about 9). Wing V produced the lowest L/D ratio. It may be recalled that the increases in C_D at low values of C_L can be reduced by trailing-edge modification.[4] Some drag reduction devices such as grooves can also reduce C_D and hence improve the L/D ratio.[7]

Computation of Potential Flow Past Airfoils

To examine the generation of extra lift by the wings with blunt trailing edges, the potential flow past airfoils with blunt trailing edges was computed using the surface vorticity technique of Martensen as simplified by Lewis.[8] The computer program is described in Ref. 9. As a first step, C_L values for different angles of attack were computed for the NACA 0012 airfoil. They were found to compare well with other published results. Then the program was applied to airfoils with 5, 10, 15, ..., 50% chord cutoffs from the trailing edge. To avoid numerical difficulty, the trailing edges were slighty rounded off as shown in Fig. 5. The stagnation point was specified to be at the middle of the rounded-off portion. Thirty-two control points were used. From the values of C_L at different angles of attack, the slopes $(dC_L/d\alpha)_0$ were calculated for the airfoils. From these, the $dC_L/d\alpha$ for the wings were calculated using formulas given in standard textbooks. The slopes for wings and airfoils are given in Table 1. The experimental values of $dC_L/d\alpha$ are lower than the theoretical values by about 25%. This is due to the fact that the theoretical values of $dC_L/d\alpha$ are realized only at R_c of the order of 10^7, whereas the R_c in the present experiments is 4×10^5. Reference 10 gives the values of $(dC_L/d\alpha)_0$ for NACA 0012 airfoils at different Reynolds numbers. The value given therein for $R_c = 7\times10^5$, when corrected for finite aspect ratio, gives a value that is very close that from the present experiments.

The computed results also show that $dC_L/d\alpha$ increases for wings with cutoff trailing edges. The percentage increase in $dC_L/d\alpha$, compared to the sharp trailing edge, is not much differnt in experiment and theory for cutoffs up to 15%. Theoretically, the increase in $dC_L/d\alpha$ is found even when the cutoff extends to 50%. The $(dC_L/d\alpha)_0$ for airfoils with 30, 40, and 50% cutoff are 0.129, 0.133, and 0.138, respectively.

However, the present experimental results at R_c of 4×10^5 show that after the 15% cutoff the increase in $(dC_L/d\alpha)$ decreases. Thus, in real fluid flow, there is an optimum value of cutoff beyond which the beneficial effects of the increases in $dC_L/d\alpha$ and $C_{L\max}$ lessen. The optimum value of cutoff at higher R_c needs to be investigated.

Conclusions

The following conclusions can be drawn from the above investigation in which the bluntness at the trailing edge of a wing having an NACA 0012 airfoil is increased by cutting off the wing by 5, 10, ..., 50% of the chord from the trailing edge:

1) When the cutoff was increased progressively from 0 to 50% of the chord, the potential flow theory shows that $dC_L/d\alpha$ increases from 0.1195 to 0.138.

2) From the experimental investigation at $R_c = 4\times10^5$, one finds that when the bluntness is small, as obtained by cutting off only 5%, the changes in $dC_L/d\alpha$, $C_{L\max}$, minimum drag coefficient, and maximum L/D are small as compared to a wing with a sharp trailing edge. Thus, a slightly blunt trailing edge, which may make fabrication simpler, may not affect the aerodynamic characteristics significantly. Also, when bluntness is increased by further reducing the chord, $C_{L\max}$ and $dC_L/d\alpha$ increase for a trailing-edge cutoff up to 15%. Beyond this, these qualities decrease. The L/D ratio decreases with bluntness.

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